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Saturation-Based Querying Procedures for the Clique-Guarded Negation Fragment

Sen Zheng, Renate A. Schmidt

June 25, 2023

The University of Manchester
The Decision Problem in First-Order Logic (DPFO2023)

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Saturation is one of the major techniques for automated theorem proving … computes the closure of a given set of formulas under resolution-based inferences

Motivation

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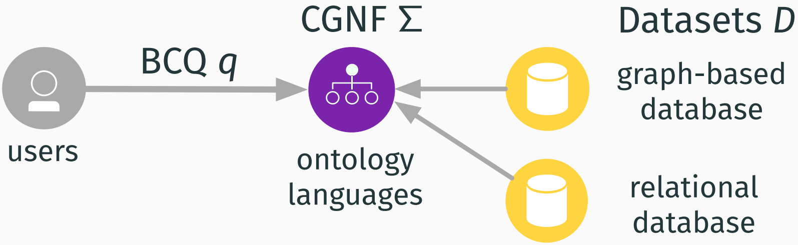

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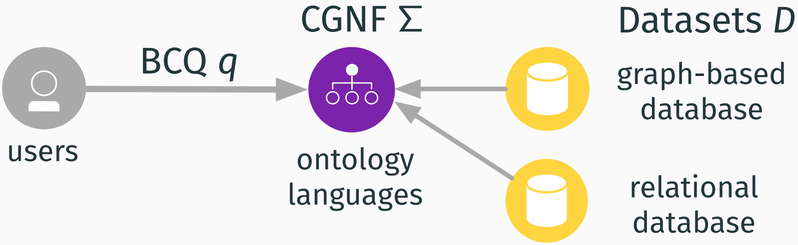

Deciding $D \cup \Sigma \models q$ to deciding unsat. of $D \cup \Sigma \cup \{\neg q\}$

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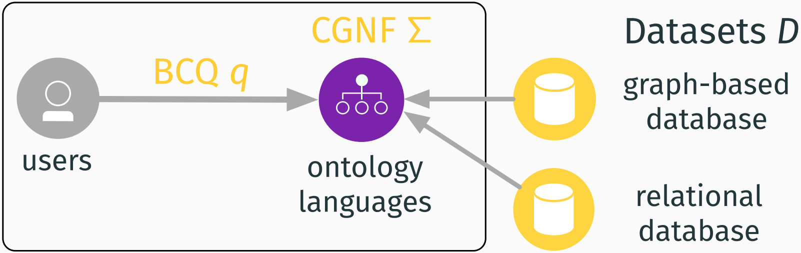

Deciding $D \cup \Sigma \models q$ to deciding unsat. of $D \cup \Sigma \cup \{\neg q\}$

1. Saturating $\Sigma \cup \{\neg q\}$ first; reusable for different D_i
2. Back-translating the saturation to a FO formula for other reasoning methods

The Guarded Fragments

guarded fragment (GF)	guarded quantification fragments
loosely guarded fragment (LGF)	
clique-guarded fragment (CGF)	

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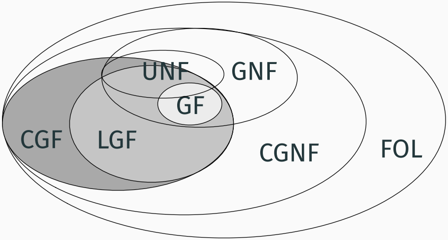

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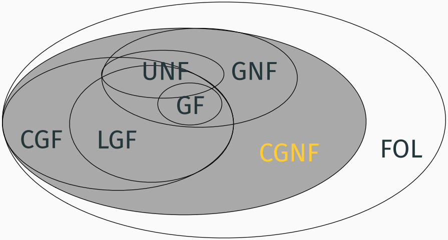
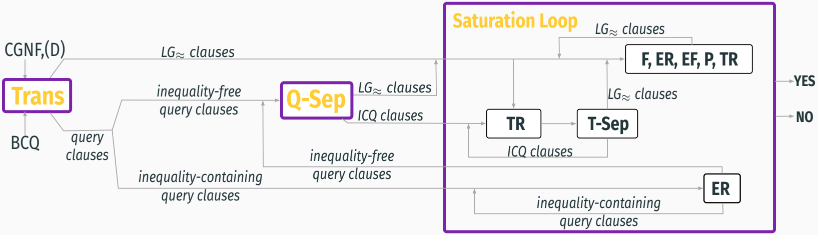


Saturation-based method for deciding $D \cup \Sigma \models q$



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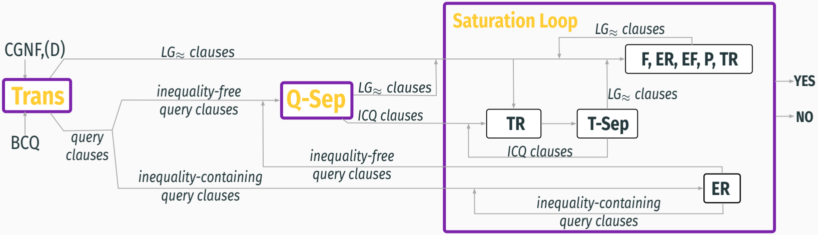
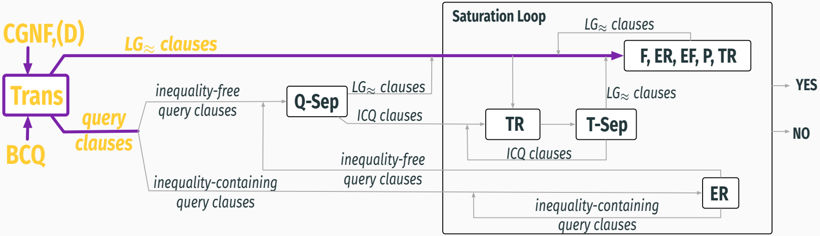


- **Trans**: Customised clausification
- **Q-Sep**: Separating (Simplifying) queries
- **Saturation Loop**: Inferences

Clausification Trans



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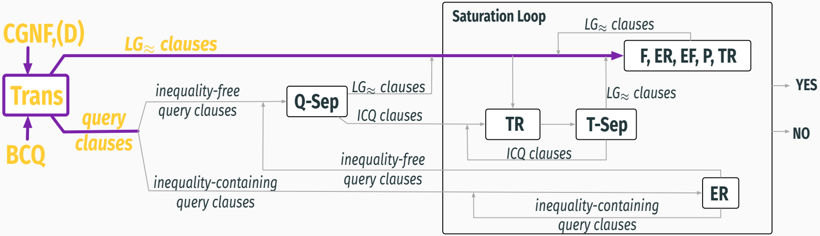

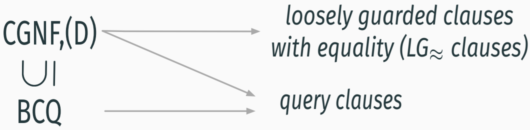

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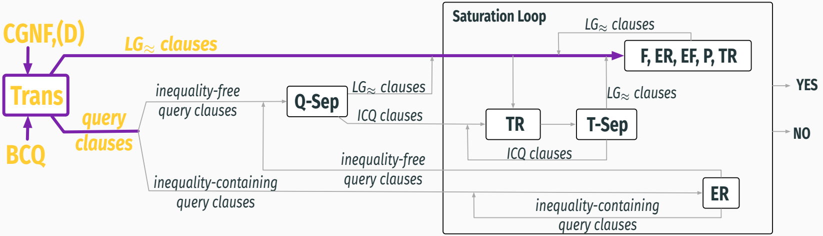

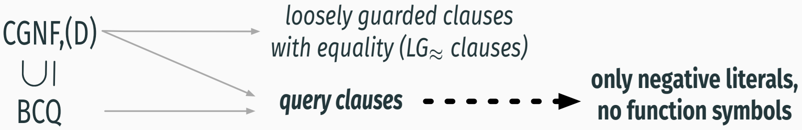

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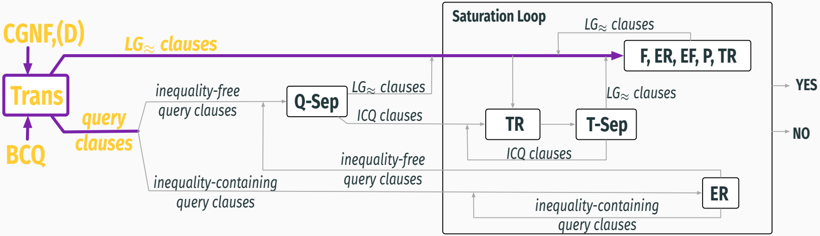
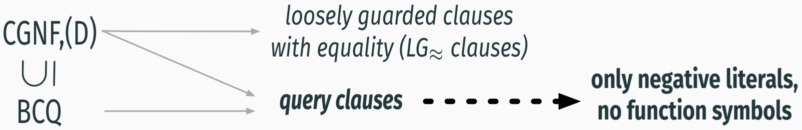
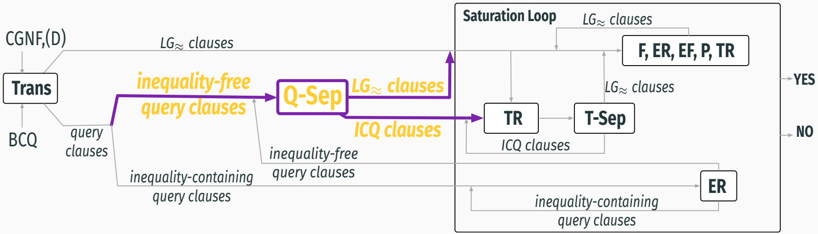




Trans reduces BCQ answering for CGNF to deciding $LG_{\approx}$ and query clauses

Separating Query Clauses Q-Sep



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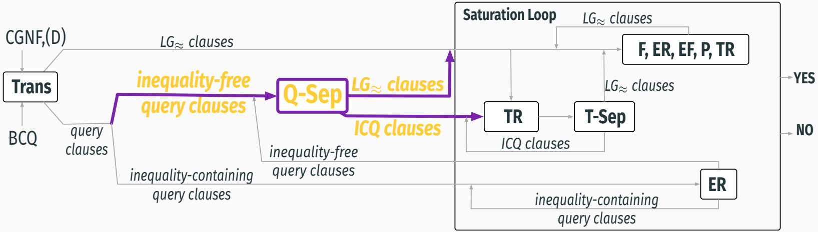

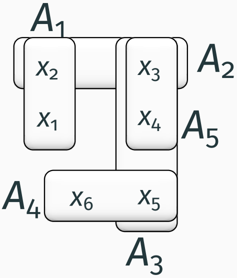

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$$\neg A_5(x_3, x_4)$$

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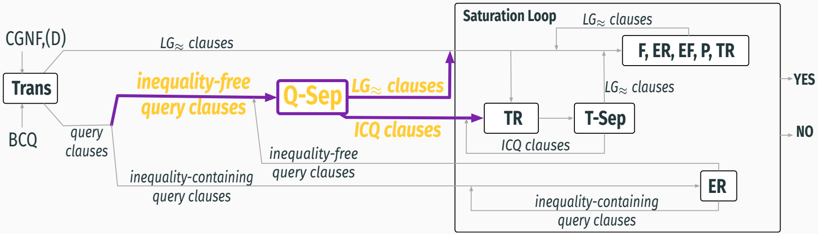

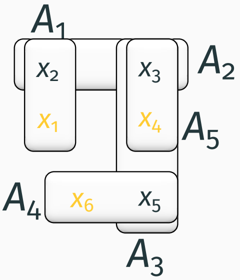

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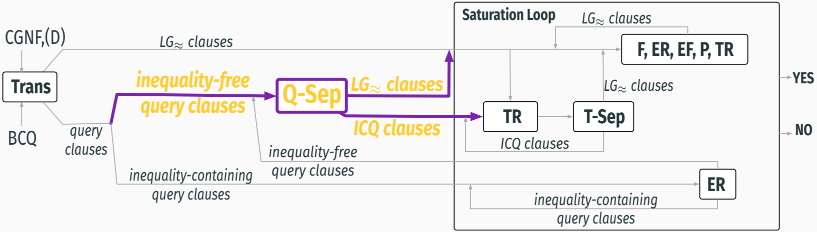
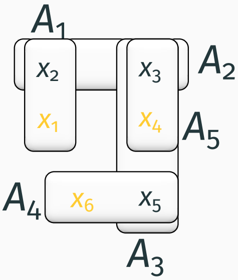
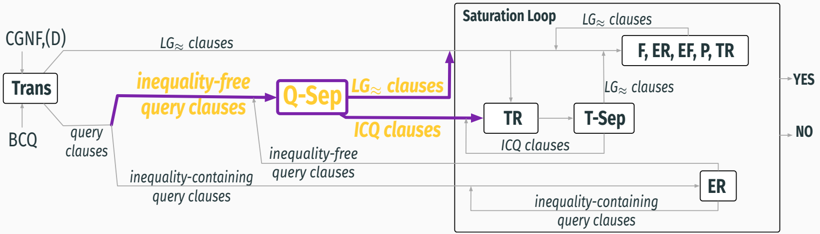
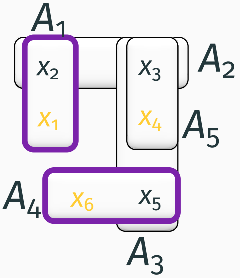




- Chained variables: x_2, x_3, x_5
- Isolated variables: x_1, x_4, x_6

Separating Query Clauses Q-Sep





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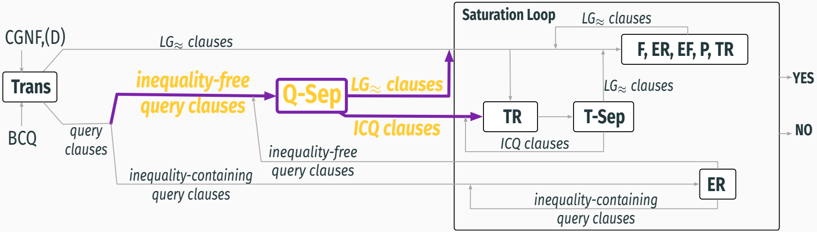
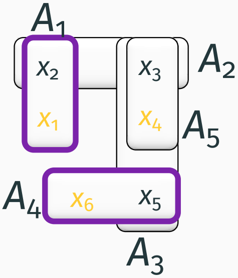
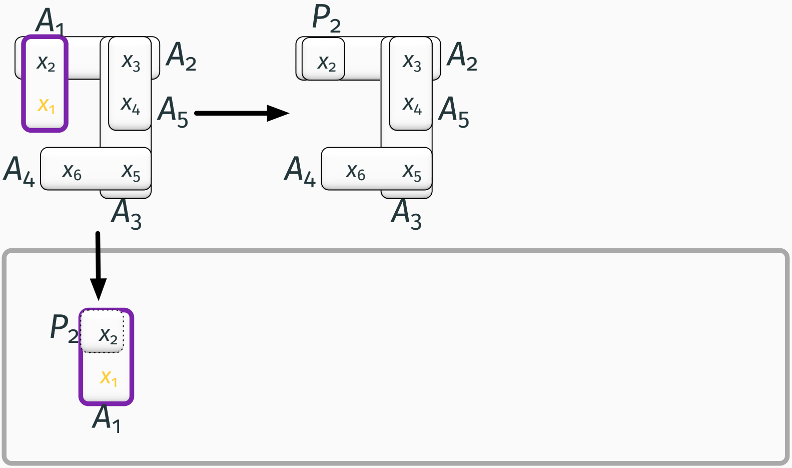




Cutting off branches!

Separating Query Clauses Q-Sep



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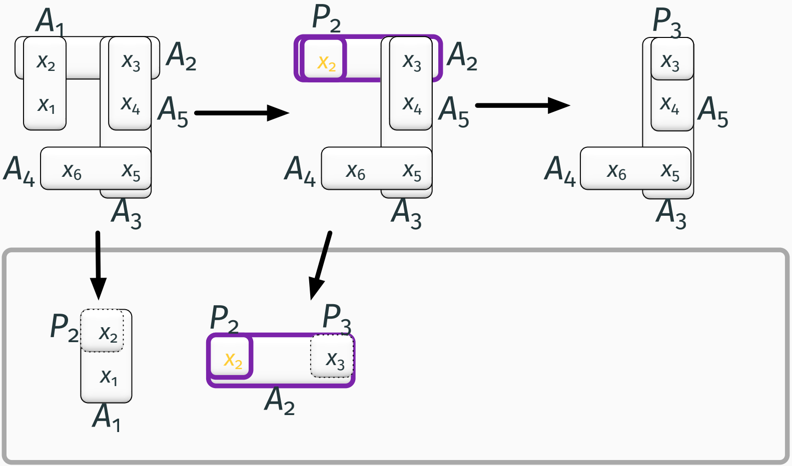
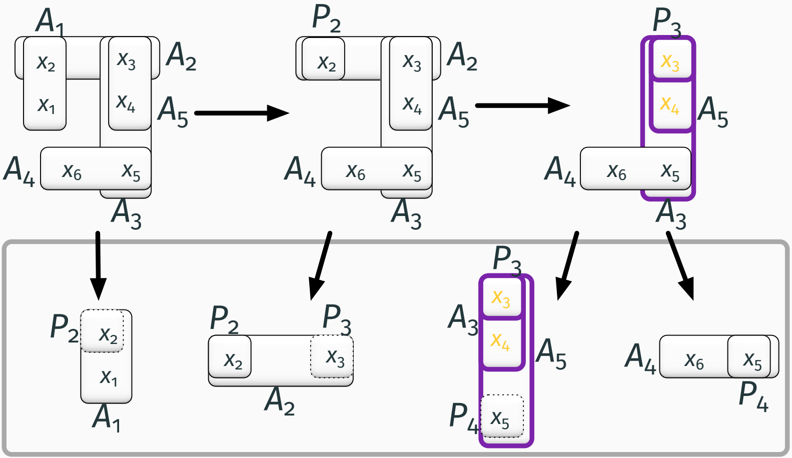


Query Simplification Q-Sep



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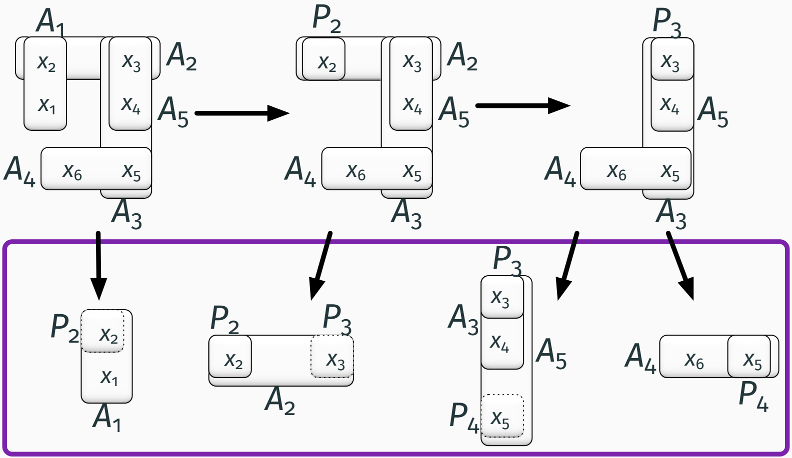

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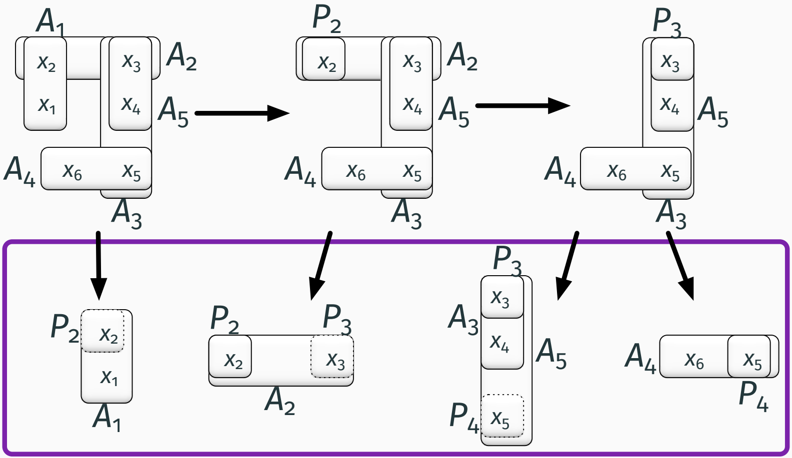

The separated clauses are $\text{LG}_{\approx}$ clauses

Separating Query Clauses Q-Sep

$$\neg A_1(x_1, x_2, x_3) \vee \neg A_2(x_3, x_4, x_5) \vee \neg A_3(x_5, x_6, x_7) \vee \neg A_4(x_1, x_7, x_8) \vee \neg A_5(x_3, x_4, x_9)$$

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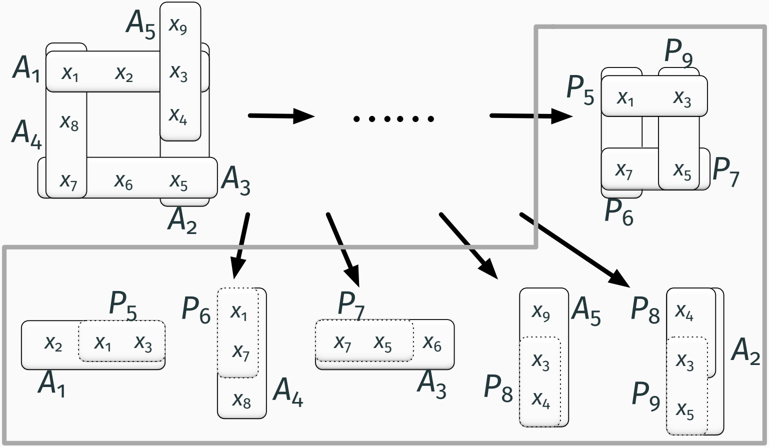

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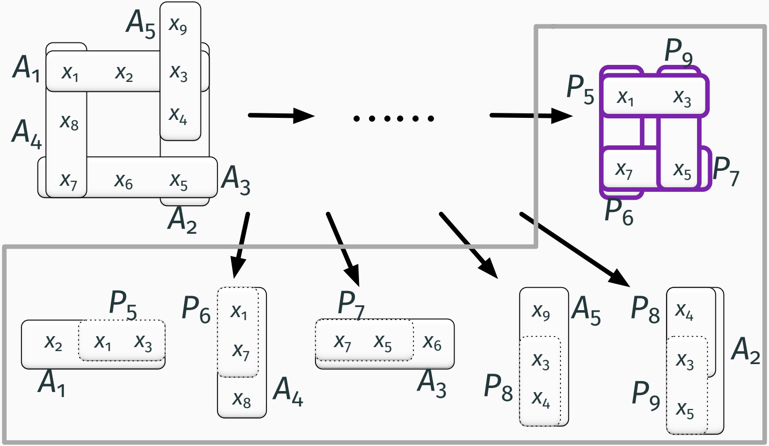

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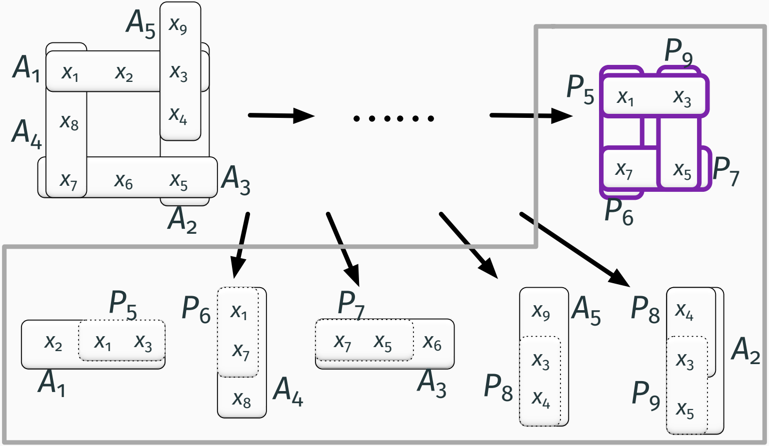
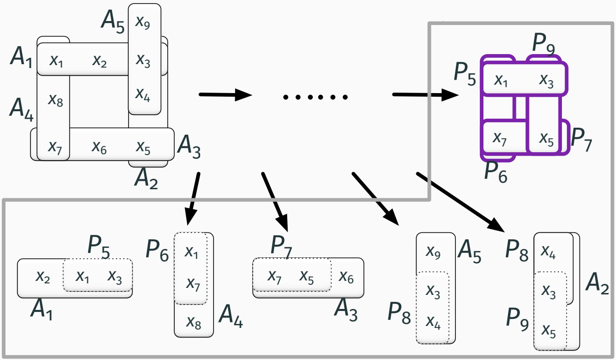


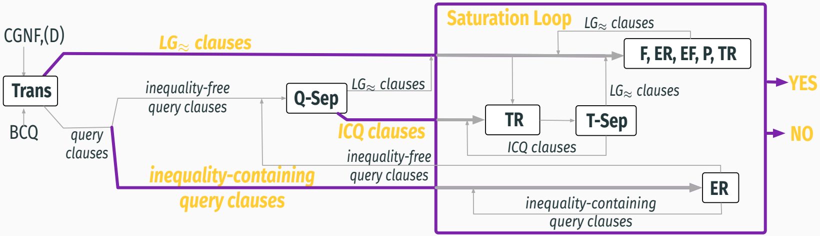
Indecomposable Chained-only Query (ICQ) clauses

Separating Query Clauses Q-Sep

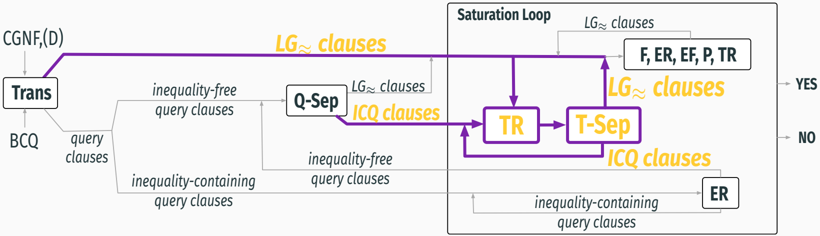


Q-Sep replaces a query clause by $LG_{\approx}$ and ICQ clauses

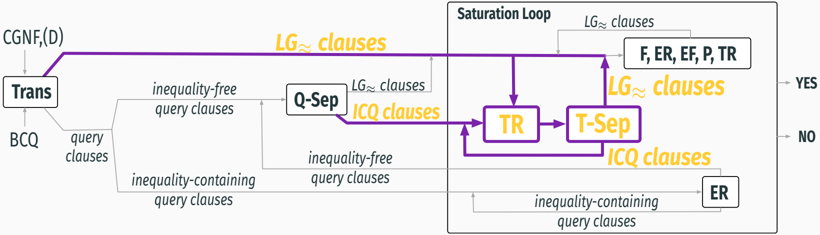
Saturation Loop

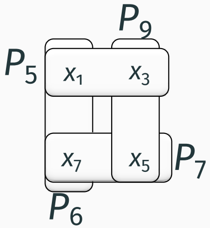


Top-Variable Resolution TR



Top-Variable Resolution TR





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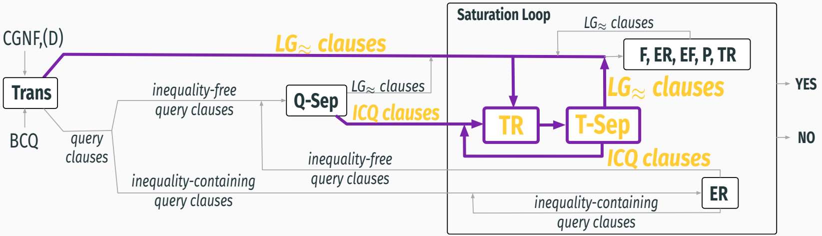

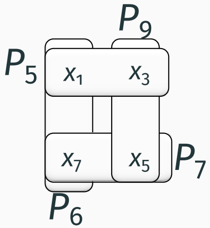

$$\neg P_5(x_1, x_3) \vee \neg P_9(x_3, x_5) \vee$$
$$\neg P_7(x_5, x_7) \vee \neg P_6(x_1, x_7)$$

TR Example

$$Q = \neg P_5(x_1, x_3) \vee \neg P_9(x_3, x_5) \vee \neg P_7(x_5, x_7) \vee \neg P_6(x_1, x_7),$$
$$C_1 = P_5(x, g(x)) \vee \neg G_1(x),$$
$$C_2 = P_9(g(y), y) \vee A(h(y)) \vee \neg G_2(y),$$
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Lexicographical Path Ordering:
$f \succ g \succ h \succ P_5 \succ P_6 \succ P_7 \succ P_9 \succ A \succ G_1 \succ G_2 \succ G_3 \succ G_4$

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Lexicographical Path Ordering:
$f \succ g \succ h \succ P_5 \succ P_6 \succ P_7 \succ P_9 \succ A \succ G_1 \succ G_2 \succ G_3 \succ G_4$

TR Example

$$Q = \neg P_5(x_1, x_3) \vee \neg P_9(x_3, x_5) \vee \neg P_7(x_5, x_7) \vee \neg P_6(x_1, x_7),$$
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- $y \rightarrow f(x)$, hence $A(h(f(x)))$ occurs in the conclusion

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- $x_5 \to f(x)$
- $y \to f(x)$, hence $A(h(f(x)))$ occurs in the conclusion
- $\{x_1 \to f(x), x_3 \to g(f(x)), x_5 \to f(x), x_7 \to x\}$

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- No nested term occurs!
- Make the resolution on $Q, C_1, \ldots, C_4$ redundant

Top-Variable Resolution TR

In an inference:

positive premises → $C_1, \dots, C_n$

negative premise → C

$$\frac{C_1, \dots, C_n \qquad C}{R}$$

Top-Variable Resolution TR

Two inferences I and I'

$$I : \frac{C_1, \ldots, C_n \qquad C}{R} \qquad\qquad I' : \frac{C_1, \ldots, C_m \qquad C}{R'} \ (m < n)$$

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- Only resolving the positive premises where unification peaks

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- I' makes I redundant since
  - $C \succ R'$, by orderings
  - $C_1, \dots, C_n, R' \models R$, by resolution
  - Premises in $C_1, \dots, C_n, R' \models R$ are smaller than these in $C_1, \dots, C_n, C \models R$
- Only resolving the positive premises where unification peaks

Applying **TR** (and **T-Sep**) to ICQ clauses and $\mathrm{LG}_{\approx}$ clauses derives $\mathrm{LG}_{\approx}$ and ICQ clauses

Back-translatable Saturations

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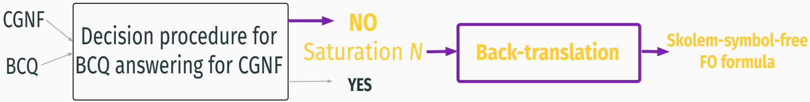
what: Eliminate Skolem symbols in N

- in general undecidable

why: Prepare N for other reasoning methods in deciding $D_i \cup N$

how: Align arguments (variables) that are under the same function symbol across clauses

Back-translatable Saturations



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Saturation-based BCQ rewriting!

Chase Example

- Well-established query answering algorithm
- Many variations

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Simplified example from [Krötzsch et al., ICDT'19]: use standard chase to decide guarded Datalog$^{\pm}$ and data

$$D = \{\mathit{Bicycle}(c)\}$$

$$\Sigma = \{\mathit{Bicycle}(x) \to \exists v.\mathit{hasPart}(x, v) \wedge \mathit{Wheel}(v), \quad \mathit{Wheel}(x) \to \exists w.\mathit{properPartOf}(x, w) \wedge \mathit{Bicycle}(w)\}$$

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$$D_1 = D \cup \{hasPart(c, n_1), Wheel(n_1)\}$$
$$D_2 = D_1 \cup \{properPartOf(n_1, n_2), Bicycle(n_2)\}$$

...

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...

May not terminate!

Saturation for the Chase Example

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$Bicycle(c)$,

$\neg Bicycle(x) \vee hasPart(x, f(x))$,

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Lexicographical Path Ordering:
$f \succ g \succ c \succ hasPart \succ properPartOf \succ Wheel \succ Bicycle$

Saturation for the Chase Example

$$
\begin{aligned}
&Bicycle(c),\\
&\neg Bicycle(x) \lor hasPart(x, f(x)),\\
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$$

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Lexicographical Path Ordering:
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Saturated; $D \cup \Sigma$ is **satisfiable**

Back-translate the Saturation

$\neg Bicycle(x) \vee hasPart(x, f(x))$,

$\neg Bicycle(x) \vee Wheel(f(x))$,

$\neg Wheel(x) \vee properPartOf(x, g(x))$,

$\neg Wheel(x) \vee Bicycle(g(x))$

$$\forall x \exists v ((\neg Bicycle(x) \vee hasPart(x, v)) \wedge$$
$$(\neg Bicycle(x) \vee Wheel(v))) \wedge$$
$$\forall x \exists w ((\neg Wheel(x) \vee properPartOf(x, w)) \wedge$$
$$(\neg Wheel(x) \vee Bicycle(w)))$$

Back-translate the Saturation

$$\neg Bicycle(x) \lor hasPart(x, f(x)),$$
$$\neg Bicycle(x) \lor Wheel(f(x)),$$
$$\neg Wheel(x) \lor properPartOf(x, g(x)),$$
$$\neg Wheel(x) \lor Bicycle(g(x))$$

$$\forall x(Bicycle(x) \rightarrow \exists v(hasPart(x, v) \land Wheel(v)))\land$$
$$\forall x(Wheel(x) \rightarrow \exists w(properPartOf(x, w) \land Bicycle(w)))$$

Can be complicated if inferences are performed!

Conclusions and Future Work

- First automated decision method for BCQ answering for CGNF
- First saturation-based BCQ rewriting method for implementing ontology-based data access over CGNF and its subfragment
- Back-translation of rewriting allows alternative reasoning methods to be used for ontology-based data access

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- Implement and evaluate the procedures
- Complexity analysis
- Query other fragments, e.g., triguarded, guarded adjacent fragment
- Retrieve non-Boolean answers

Thank you. Questions?