Deciding the Loosely Guarded Fragment and Querying Its Horn Fragment Using Resolution

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Aim

1. Deciding LGF

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The loosely guarded fragment (LGF)

- Subsumes the guarded fragment
- Subsumes description logic $\mathcal{ALCHIO}$
- Generalises standard modal logics $\mathcal{K}, \mathcal{D}, \mathcal{S}4, \mathcal{B}$
- Express the temporal logic operater **until**

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Deciding LGF

1. $\Sigma \models \bot$, given Σ in LGF

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The Horn loosely guarded fragment (Horn LGF)

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BCQ answering for Horn LGF

2. $\Sigma \cup \mathcal{D} \models q$, given Σ in Horn LGF, ground atoms $\mathcal{D}$, a BCQ q

Motivation

Deciding LGF/BCQ answering for Horn LGF

- Hyper-tree width queries $\mathcal{D} \models q$
- Query containment/evaluation/entailment $\Sigma \cup \mathcal{D} \models q$
- Constraint-satisfaction/homomorphism problem
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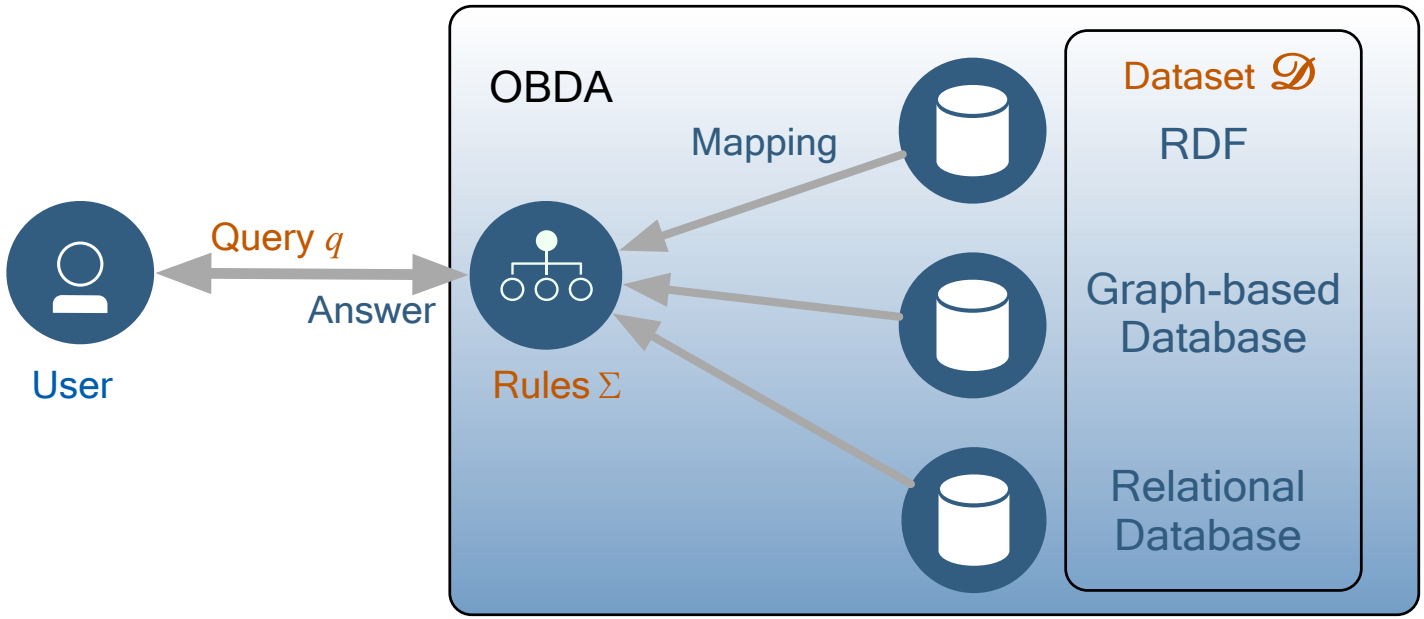

Problems of interest

Some decidability results:

- First-order logic is undecidable
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Contributions:

1. A practical procedure for deciding LGF
2. First practical procedure to query Horn LGF
3. First practical procedure for answering star queries and cloud queries over LGF

Resolution decides LGF

Shown by [de Nivelle et al. 2003] and [Ganzinger et al. 1999]:

- Introduce the 'MAXVAR' technique

Problems:

- Complicated
- Not considering queries

Reasoning using resolution

1. Deciding LGF: $\Sigma \models \bot$
2. Querying Horn LGF: $\Sigma \cup \mathcal{D} \models q \quad \Leftrightarrow \quad \Sigma \cup \mathcal{D} \cup \neg q \models \bot$

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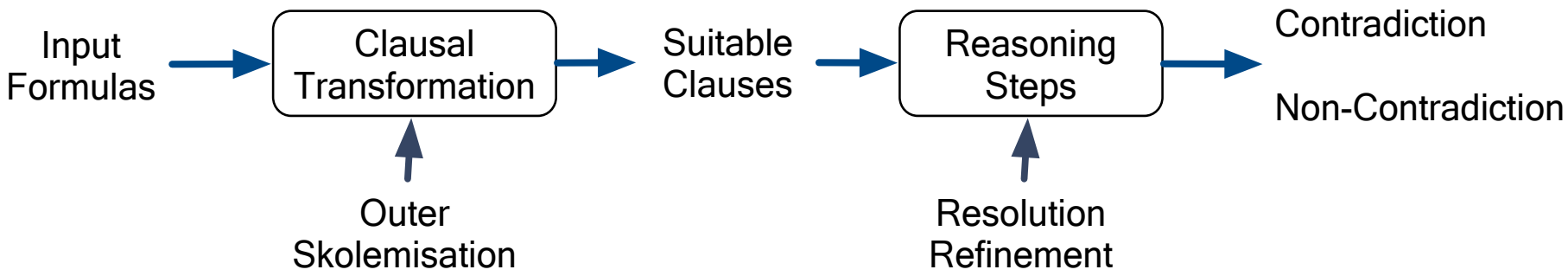

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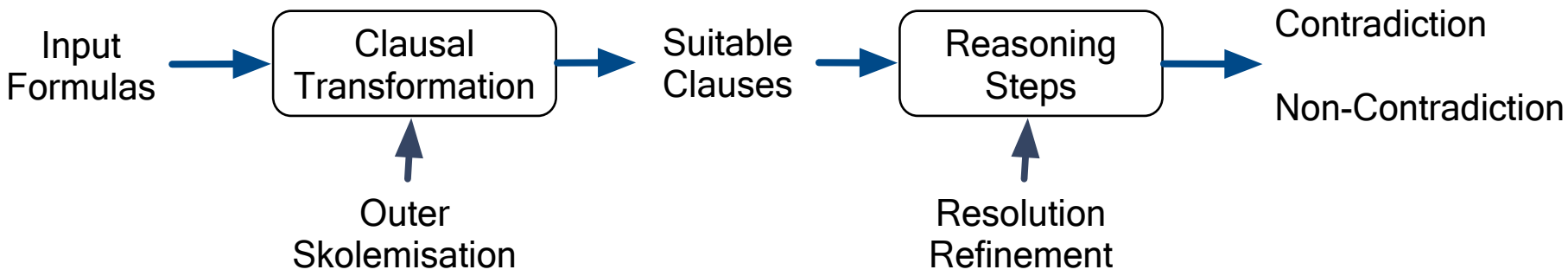

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In finitely many steps!

Deciding LGF using resolution

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Deeper terms and wider resolvents

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Example 1

Given loosely guarded clauses C, C_1, C_2 and C_3:

$$C = \neg A_1 xy \vee \neg A_2 yz \vee \neg A_3 zx \qquad C_1 = A_1(fx, x) \vee D(gx) \vee \neg G_1 x$$
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C, C_1, C_2 and C_3 derives $D(g(fx)) \vee \neg G_1(fx) \vee \neg G_2 x \vee \neg G_3(fx)$.

Contribution 1: Deciding LGF

LGF-Res system: ordered resolution with selection using **a special top variable selection refinement**

- A variation of [de Nivelle et al., 2003]
- No need for a specific unification algorithm
- Within the framework of [Bachmair et al., 2001]
- Fully developed definitions and proofs
- Minor corrections

Our methods for deciding LGF

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1. $\{x/ffx', y/fx', z/fx'\}$ to substitute variables in C
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No term depth increase!

Contribution 2: Querying (Horn) LGF

Query-Res system: extends *LGF-Res* by **considering queries**

- Compute top variables in query clauses
- Query pair clauses $\Rightarrow$ not limited to LGF
- Top variable resolution $\Rightarrow$ rewriting queries
- Querying Horn LGF
- Star/cloud querying over LGF

Conclusions and Future Work

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- Querying the whole of the (loosely) guarded fragment?
- Querying the guarded negation fragment ($\approx$)?
- Deciding fluted logic?
- Experiments

Thanks!