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Deciding the Loosely Guarded Fragment and Querying Its Horn Fragment Using Resolution

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Aim

1. Deciding any set of loosely guarded formulas Σ.
2. Given a Boolean conjunctive query q, a set of Horn loosely guarded formulas Σ and a set of ground facts $\mathcal{D}$, apply resolution to decide $\Sigma \cup \mathcal{D} \models q$, by checking satisfiability of $\Sigma \cup \mathcal{D} \cup \neg q$.

The loosely guarded fragment (LGF):

- Can be decided in 2EXPTIME
- Extends the guarded fragment
- Extends $\mathcal{ALCHIO}$ and $\mathcal{K}, \mathcal{D}, \mathcal{S}3, \mathcal{B}$
- Horn LGF extends the guarded existential rules

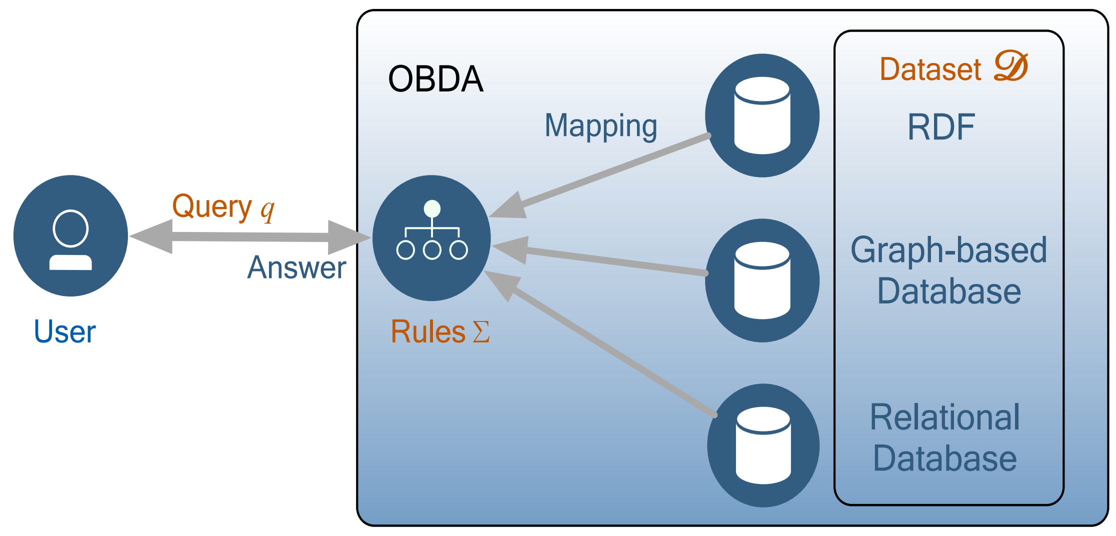
Boolean conjunctive queries (BCQs):

- Returns yes or no

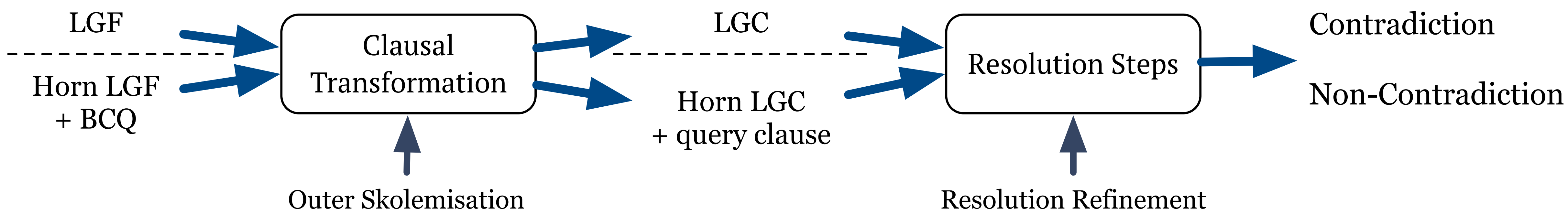
As yet no practical procedure to query Horn LGF.

Motivations

- Hyper-tree width queries are in LGF
- Query evaluation/entailment/containment
- Constraint-satisfaction/homomorphism problem
- Ontology-based data access (OBDA) systems



Decision Procedures



Clausal Transformation

Negate the BCQ.

$\exists xyz(Axy \wedge Byz) \Rightarrow \neg Axy \vee \neg Byz$.

Use *LGF-Trans*:

- Optimised structural transformation
- Prenex normal form
- Outer Skolemisation

From $\forall xy(Axy \rightarrow \exists w Bxw)$ to $\neg Axy \vee B(x, fxy)$

Clausal Forms

Query clauses:

- Negative
- Only variables and constants

Loosely guarded clauses (LGC) C:

- (Horn), simple and weakly covering
- Each pair of variables in C co-occurs in a negative flat literal of C

Resolution Refinement

Use an admissible ordering with a precedence s.t. **function symbols** $>$ **constants** $>$ **predicate symbols**. Eligible literals L in a clause C is found by:

```
if C is ground then
  | L = Maximal(C)
else if C has a negative compound literal L then
  | L = Select(C)
else if C has positive compound terms then
  | L = Maximal(C)
else
  | L = SelectTop(C)
```

$SelectTop(C)$ **is performed in two steps:**

1. Select all negative literals in C, and
2. if C contains top variables, select negative literals in C containing top variables.

Results

In resolution steps, no variable depth increase occurs and no infinite length increase occurs.

1. *Query-Res* decides LGF.
2. *Query-Res* decides the problem of BCQ answering for Horn LGF.
3. *Query-Res* decides the problem of loosely guarded query and star/cloud query answering for LGF.

Examples

Example 1: Deciding LGCs

$C_1 = \boxed{\neg A_1xy} \vee \boxed{\neg A_2yz} \vee \boxed{\neg A_3zx}$ $\quad C_2 = A_3(x, fx)^* \vee \neg G_3x$

$C_3 = A_2(fx, fx)^* \vee \neg G_2x$ $\quad C_4 = A_1(fx, x)^* \vee D(gx) \vee \neg G_1x$

$C_5 = \boxed{\neg Dx}$ $\quad C_6 = G_1(fa)^*$ $\quad C_7 = G_3(fa)^*$ $\quad C_8 = G_2a^*$

Assume $f > g > a > A_1 > A_2 > A_3 > D > G_1 > G_2 > G_3$. Start reasoning with C_1. Applying resolution to C_1, C_2, C_3 and C_4 produces an mgu $\{x/ffx', y/fx', z/fx'\}$ to substitute variables in C_1. Hence x is the top variable in C_1. $\neg A_1xy$ and $\neg A_3zx$ in C_1 are selected. Applying resolution to C_1, C_2 and C_4 derives $C_9 = \neg A_2xx \vee D(gx)^* \vee \neg G_1x \vee \neg G_3x$. Applying resolution to C_9 and C_6 derives $C_{10} = \boxed{\neg A_2xx} \vee \boxed{\neg G_1x} \vee \boxed{\neg G_3x}$. x is the top variable in C_{10}. Apply resolution on C_3, C_6, C_7 and C_{10} derives $\boxed{\neg G_2a}$, which derives $\perp$, by applying resolution with C_8. Hence, the given set of LGCs is unsatisfiable.

Example 2: BCQ answering for Horn LGCs

$Q = \boxed{\neg A_1xy} \vee \boxed{\neg A_2yz}$ $\quad C_1 = A_1(fxy, x)^* \vee \neg G_1xy$

$C_2 = A_2(gxy, x)^* \vee \neg G_2xy$

Q is the clausal form of a negated BCQ $q = \exists xyz(A_1xy \wedge A_2yz)$. Apply resolution to C_1, C_2 and Q derives $\neg G_1(gxy, y') \vee \neg G_2xy$, which is neither loosely guarded nor a query clause. Identifying that x is the top variable in Q, only $\neg A_1xy$ is selected in Q. Hence apply resolution to C_1 and Q derives $\boxed{\neg A_2xz} \vee \boxed{\neg G_1xy}$. Then the given set is satisfiable, thus the answer to the query q is no.

L^* means the maximal literal and $\boxed{L}$ means the selected literals.